Optical Surface Specification using the Structure Function

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Abstract: The structure function is suggested as a means of specifying surface figure and roughness in order to avoid surface errors that have regions of high slope that produce unacceptable imaging performance.
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1. Introduction

As more optical surfaces are aspheric and others are being fabricated by non-traditional means such as diamond turning or finished with MRF or computer controlled polishing, the surfaces often exhibit characteristics that make them unusable for their intended purpose yet the surfaces meet the drawing specifications. It is a case where the technology of optical finishing has gotten ahead of the understanding of how to write specifications that cover the types of surface errors these non-traditional methods introduce in surfaces.

It is not that a means of writing a specification does not exist. Many workers in the area of surface roughness [1-4], have addressed the problem but generally from a sufficiently theoretical manner that the engineers writing the drawing specifications have no idea how to use the available tools. The ISO optical drawing standard 10110 Part 7 on Texture [5], gives an example of how to use a power law description for surface roughness but the standard is not widely used nor the method well understood.

It is the purpose of this paper to show that a relatively simple method of specifying surface roughness, the structure function, can be used to assure that surfaces do not have unacceptable mid-spatial frequency roughness. Further, that optical shops with phase measuring interferometers have the tools necessary to test the surfaces they make to see if they meet the structure function specification.

2. Description of the structure function

The structure function first appears to have been used to describe how the phase of a wavefront is disturbed as it passes through the atmosphere [6] and is thus important to astronomers among others. The structure function is defined as

$$D(\underline{r}) = \langle [p(\underline{r}') - p(\underline{r}'-\underline{r})]^2 \rangle \qquad (1)$$

where p(r') is the phase at r' and p(r'-r) is the phase a distance r away from the first point, r'. The difference in phases are squared (to keep things positive) and averaged over all the measurements. D(r) is plotted against r on a log-log scale and for most traditionally polished surfaces, as well as typical atmospheric turbulence, this graph results in a straight line. This shows that the phase differences follow a power law based on the distance between the points at which the phase was measured, or to a good approximation for optical surfaces

$$D(r) = Ar^B \qquad (2)$$

This power law breaks down for real optical surfaces when the phase differences are measured over distances that are a substantial fraction of the aperture, say 10 to 100% of the full aperture because the act of polishing smoothes out the roughness. This roughness between widely spaced points in the aperture is generally considered figure error and is specified using a peak-to-valley number, or preferably, an rms value. Thus the structure function for a well figured surface might look like that in Fig. 1. The part of the curve from the most closely spaced points ~0.1% of the diameter up to 10% of the diameter are nearly a straight line and have been fit to a power law.

At this point some of the more mathematically inclined might pause to say why not use the power spectral density (PSD) to describe the surface roughness? My answer is that the structure function contains the same information as the structure function and one can be derived from the other in a mathematical sense [7]. In practice the information for calculating either method comes from a rectilinear array and this forces the calculation of the

PSD to be done in a rather artificial way, particularly for most optics that have a circular aperture. For the structure function the shape of the aperture does not matter and the calculation is independent of the azimuth in the aperture. Finally, the separation scales for the PSD are in units that are more difficult to relate to than simple distances or distance squared. Many people do not think in inverse meters, nor meters^3, units on a PSD graph. With the structure function the abscissa or x axis is in meters and the ordinate or y axis in meters^2 (or nm^2). By taking the square root of the structure function the result is the rms at that separation in meters (or nm).

3. Writing a specification using the structure function

For years the figure of optical elements has been specified in terms of a peak-to-valley or rms number. In this age of phase measuring interferometers it makes sense to specify figure as an rms value as this directly relates to image quality. At the phase difference end for widely separated points in the aperture this can still be done and the structure function will simply be the square of this. For a diffraction limited wavefront the usual specification is about λ/14 or 0.07λ or 0.045 μm rms. This gives about 2e3 nm^2at the full aperture. This sets the value of the structure function until the separation between points is somewhere between 25 and 10% of the aperture.

At the short separation end is what is commonly referred to as finish. A reasonably well polished glass surface will be or the order of 1 nm rms here. Less demanding optics might be in the range of 3 to 5 nm rms, but this part of the specification is again a part that has some historical background so there should not be a problem coming up with a suitable value for the optic under question.

The separation between measured is more of an issue. A surface roughness microscope will sample the surface every couple of μm and this is the kind of instrument used to measure finish so the couple nm rms finish should be applied at the couple of nm scale. Thus the graph for the specification would look like Fig. 2

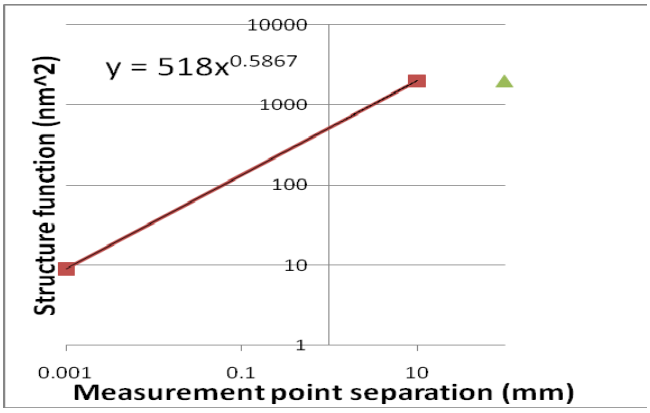

Fig. 2 Graph of a hypothetical Structure function specification

Here we have assumed an rms roughness of 3 nm at 1 μm spatial scale (or 9 nm^2 rms structure function) and an rms figure error of 45 nm rms (or ~2000 nm^2 rms) that extends from a tenth the full aperture to the full aperture. This gives a power law specification that covers the mid-spatial frequency errors from finish to figure with a power law of

$$SF = 518x^{.587} \qquad (3)$$

Where x is the separation between measured points. For those who remember their logarithms it is relatively easy to calculate the values in the power law or you can cheat and use the power trend line function in Excel as I did. Now that we have a way of writing a specification for mid-spatial frequency errors, how do we measure the structure function.

4. Measuring the structure function

Most interferometer software does not at present have a button to push for calculating the structure function of an OPD map so the OPD map must be exported to a program such as Matlab. Once the OPD map is in Matlab the

calculation of the structure function is quite simple. Using a random number generator select a pair of points withing the aperture, calculate the distance between the points and the square of the phase or height difference between the two points. Store the square of the phase difference and the distance. Now repeat this operation a large number of times. From some experimenting 10e5 to 10e6 points will give a good estimate and take perhaps a second of computer time.

Now divide the diameter of the aperture into perhaps 100 equal boxes and sort the results by the distances between points. Within each box, average the squared phase differences and plot the squared phase difference versus the distance between measurement points and you have the structure function for that interferogram. The structure function you get is an estimator in the sense that if you repeated this calculation you would get a very slightly different result since you would be using a different set of random samples in the apertrue. However, by using a large number of samples any repeated calculation will give almost the same result. Fig. 3 is a typical result of 31 interferometric tests of different areas of the same part to show that the results are repeatable within a small range .

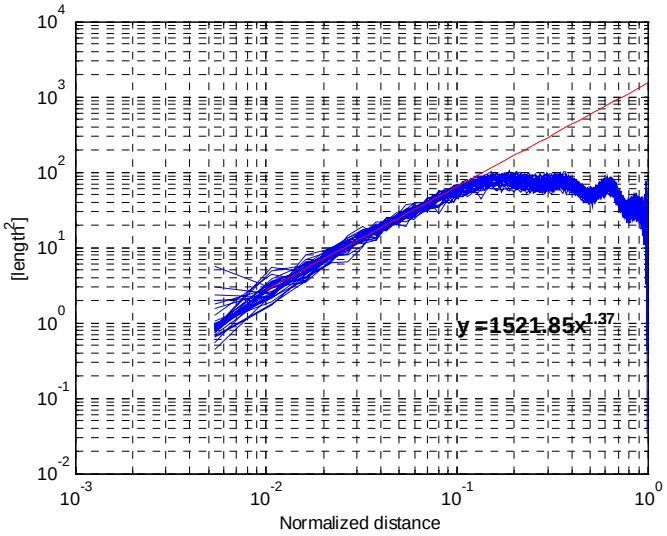

Fig. 3 Structure functions calculated from 31 interferometric measurements of the same surface

5. Conclusions

We have described the structure function and shown it has the same basic information as the power spectral density function. The definition has been used to write a specification for surface figure and finish that includes mid-spatial frequency roughness. Finally we have shown how the structure function can be calculated from an interferometer OPD map and that typical results are repeatable to a level that should be useable in a shop environment.

6. Acknowlegements

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7. References

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