Specifications: Figure and Finish are not enough

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ABSTRACT

Several examples are given of optics apparently specified only by figure and finish. Although these optics met the specifications they did not produce good images. The presumed reason for the poor performance was the lack of a specification for mid-spatial frequency roughness. We show that a reasonable specification can be applied using the concept of a structure function, a mathematically simple function easily calculated from interferometric phase data at each pixel. An example wavefront is used to show how the specification can be developed from typical figure and finish specifications and include information about roughness in the mid-spatial frequency region.

Keywords: Structure function, mid-spatial frequency roughness, narrow angle scatter, interferometry, finish

1. INTRODUCTION

This paper is about several instances of failure to consider the effects of mid-spatial frequency roughness on optical performance and wondering if the lessons will ever be learned. My first experience was anecdotal but the person responsible was clearly upset by the results so I assumed perhaps the word had gone out; this was some 30 or more years ago. In the meantime I participated in some optical standards discussions about figure and roughness, and a section was included in the ISO 10110 optical standard for specifying the power spectral density (PSD) from full aperture down to sub-millimeter spatial scales; this standard was published about 10 years ago.

Thus it was with some surprise, when I was asked to demonstrate an alignment microscope marketed by my company, that it was obvious that someone had apparently overlooked mid-spatial frequency figure errors in the optics used in the demo. However, with this result clearly implanted in my mind another client who was using my microscope complained that after using all his skill he could not get his collimator to produce a nice, clean Airy pattern on axis. At my request he sent a picture of the image and I had to tell him that it was doubtful that he ever would get a tight Airy pattern with the optics he had.

This paper will review the above instances of the problem with mid-spatial frequency roughness and suggest a clearer way of specifying surface topography that does not use fancy math and is fairly transparent about the meaning of the numbers involved; use of what the astronomers call the structure function. The paper concludes by suggesting that there is a relatively simple method of specifying mid-spatial frequency roughness and describes a way of testing the finished optic to see that it meets the specification.

2. UNHAPPY EXPERIENCES

2.1 First sad experience

At least thirty years ago when I was managing the optics shop at what was then known as the Optical Sciences Center (and is now the College of Optical Sciences) at the University of Arizona, we had a Government contract monitor come by. I am not sure who else he may have talked to at Optical Sciences, but he told me about ordering a large, reasonably fast for the day, parabola that he hinted was destined for use in a laser weapons system, and how when it was tested it was completely unacceptable because the wavefront was full of very low amplitude, yet small in area relative to the diameter of the mirror, hills and valleys. What was so distressing to this man was that this very expensive mirror met both the rms figure error spec and the rms finish spec, but was going to be useless for its intended purpose. There was nothing we could do to help but the distress of this man made a lasting impression.

2.2 Work on a possible solution

For several years in the 1980's and early 90's I was part of the US delegation to the ISO Technical Committee 172 for Optics and Optical Instruments. My particular area of involvement was in the area of how to put specifications on optical drawings. This work eventually led to the publication of ISO 10110-Preparation of optical drawings[1]. One of the sections, Part 7, dealt with surface texture and included a means of specifying the power spectral density of the texture of optical surfaces. This designation was included because some of the delegates, mainly those dealing with laser optics, insisted such a specification had to be included despite the fact that many of the delegates did not understand what the power spectral density (PSD) was, how it was calculated from an interferogram of the surface, or how to write a meaningful specification using the concept of the PSD. On the other hand it seemed to me that a great step forward had been made by at least providing a means of specifying mid-spatial frequency roughness and that now the more general practitioners in optics would begin using the new tools available.

2.3 Contemporary sad experience 1

While maybe the laser community understands the importance of mid-spatial frequency roughness (your optics can disintegrate if the roughness is not controlled), parts of the imaging community do not yet appreciate that specifying just figure and finish are not enough to produce a high performance system. To back up this statement I give a little history.

My company makes an autostigmatic microscope called the Point Source Microscope (PSM) that forms a near perfect spot of light at the focus of the objective and can re-image that spot on a CCD camera when a wavefront returns after being reflected from an optic or optical system conjugate with the focus. One day, we got an invitation to demonstrate the PSM from a company that had an assembled optical system and wanted to see how the PSM worked when looking at the system in autocollimation. Naturally, we accepted because this company was potentially a good customer.

Since the optical system they wanted to use in the demo was proprietary there was not too much I learned about the system itself except that it was somewhat complex. They asked me to put the PSM at the system focus while they put a good plane mirror in front of the system to autocollimate it. The return spot, while not perfectly round, was nice and tight as shown in the intensity contour map made from a Matlab analysis of the stored PSM png image seen in Fig. 1 left. The FWHM of the spot is about 7 x 10 pixels and with the magnification of the PSM with a 10x objective the single pass spot size for the system under test was about 3 x 5 µm, very respectable for a system with several elements. It should be noted that the PSM exposure was adjusted so the brightest pixel was just under the 8-bit saturation level.

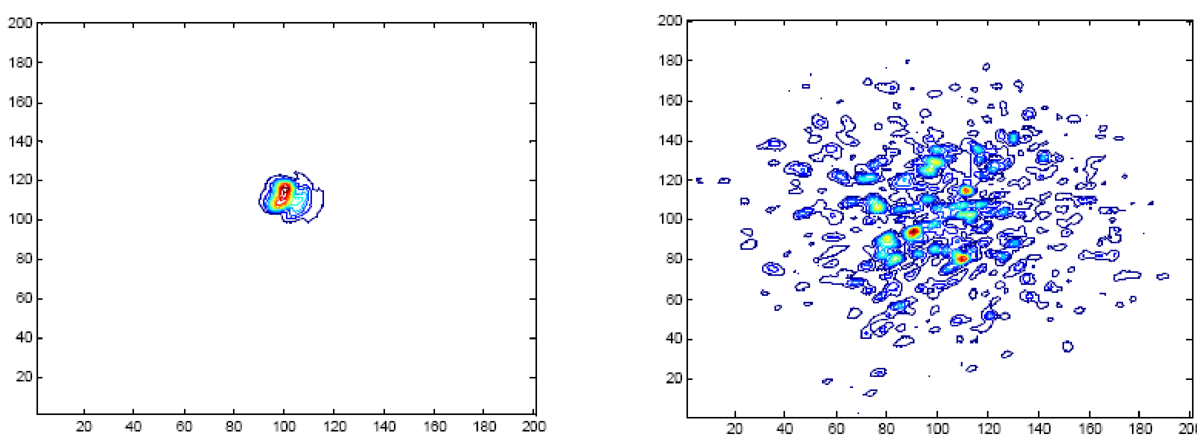

Fig. 1 Image of the first system tested in autocollimation with a FWHM of about 3 x 5 µm (left). Image of the second identical system tested with a FWHM of about 20 µm (right). The units are detector pixels and the most intense pixels are just under saturation in both cases although the exposure in the right hand image is about 50 times longer than in the left.

Then I was asked to put the PSM at the focus of a second system which was claimed to be identical except the optical elements were made by a different vendor. After substantially increasing the exposure it was finally possible to find the return spot from this second system but it looked completely different from the first as seen in Fig 1right. Again the brightest pixel was just under saturation and a crude estimate of the FWHM is about 20 µm and the exposure was about 50 times that of the first system to get the brightest pixel just under saturation. All I was told was that the optical elements met spec, but it didn't take too much imagination to guess what was wrong.

2.4 Contemporary sad experience 2

This brings me to my most recent case where a client called and said he was very frustrated because he was sure he had his system in about as good alignment as it could be, but still couldn't get a decent looking image. He offered to send a picture of the image which is poorly reproduced in Fig. 2. After looking at Fig. 1 right it is not hard to see why I suggested he would probably do no better. Again the optics met spec for figure and finish but was it a good spec?

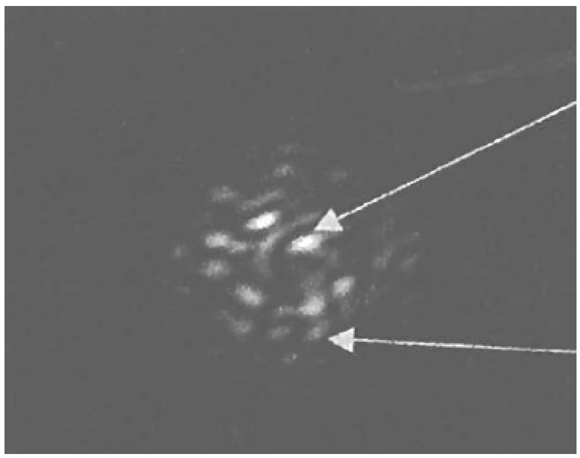

Fig. 2 Image produced by a collimator with point source illumination in autocollimation with brightest pixels at or slightly above saturation

3. SPECIFICATION BASED ON THE STRUCTURE FUNCTION

3.1 Definition of the Structure Function

There have been good methods for specifying the errors in optical surfaces as a function of spatial wavelength for many years. A number of papers have been written on the subject by members of the X-ray community as any roughness in the grazing incidence optical reflectors scatters the short wavelength X-rays enough to make an X-ray telescope or microscope worthless.[2,3] Most of these papers use the power spectral density (PSD) as the means of describing the roughness, and plot PSD in units of length cubed versus Spatial frequency in inverse length units, units that are not intuitively obvious to those outside this narrow field. Further, the papers are full of equations and terms such as auto-covariance function and Barlow windows, again terms rather foreign to many optical engineers and fabricators.

A simpler approach is that taken by astronomers in their efforts to specify the topography of large telescope mirrors so that atmospheric seeing limits their image quality rather than errors in their optics. This effort started with a model for astronomical seeing developed by Tatarski[4] and Fried[5] based on work of Kolmogorov[6] describing atmospheric turbulence. For light propagating through turbulence Kolmogorov defines the structure function $D_p(\underline{r})$ of the wavefront phase perturbations $p(\underline{r})$ as

$$D_p(\underline{r}) = \langle [p(\underline{r}') - p(\underline{r}'-\underline{r})]^2 \rangle = (\lambda/2\pi)^2 * 6.88\, (|\underline{r}|/r_0)^{5/3}$$

where r_0 is the Fried parameter and is typically about 100 mm and we have assumed a wavelength of 500 nm. This so called 5/3 law is plotted in Fig. 3 on a log/log scale.

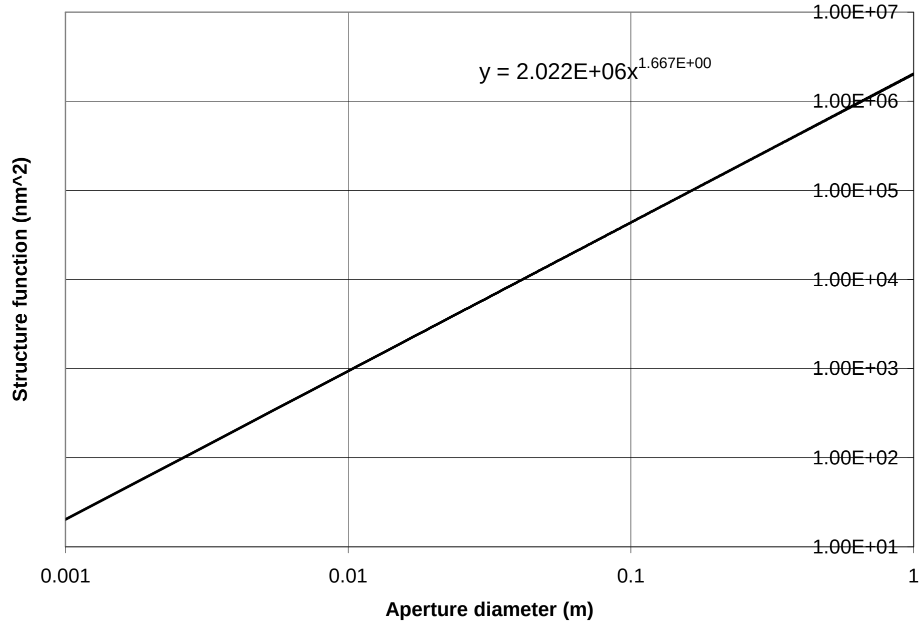

Fig. 9 The Kolmogorov turbulence structure function plotted for a Fried parameter of 0.1 m and a wavelength of 500 nm

3.2 Features of the structure function

Several things are attractive about this structure function to the less mathematically inclined. The units make sense; the structure function is a squared length so its square root, the rms, will be positive; and it is plotted against the separation of measurement points within the aperture in length units. The structure function is independent of the azimuth within the aperture where the PSD is usually thought of as being taken along a particular diameter. Finally, the structure function contains the same information as the power spectral density but in a different form.[2,7]

It is also clear how one might calculate a structure function given an interferogram of the surface of an optic. Pick some random point within the aperture and then a second random point; the distance between these points will be the r of the structure function. At each point there will be an optical phase and the difference will be a $\Delta\varphi$, the difference in phase between the points. This is squared and stored along with the corresponding r. The process of picking random points within the aperture and storing the square of the phase difference is repeated some large but sensible number of times. For example, a million points can be sampled in about 2 seconds on a common laptop.

The points are then binned and sorted by r. The $\Delta\varphi$'s associated with each r are averaged so there is an average variance in the phase for each bin. The average variance, the square of the rms phase difference, is plotted versus r to give a graph like that in Fig. 4b. The contour map of the data used, with tilt and focus removed from the interferogram, is in Fig. 4a.

3.3 Related features of the structure function

Obviously the wavefront represented in Fig. 4 is very high quality with a maximum variance of about 45 nm^2. It is also fairly clear how the data was obtained from the interferogram and how the structure function was calculated. While there are many data points used in the calculation, once the process is written in code the results can be repeated in seconds. The reason for using many data points is that with a small sampling the shape of the structure function changes each time it is calculated because what we are using is, in a statistical sense, an estimator[8] of the true influence function obtained by using all the points in the interferogram. By the time 10^4 or 10^5 samples are taken in a field of around $6x10^5$ total points, the structure function varies less than a couple percent from calculation to calculation.

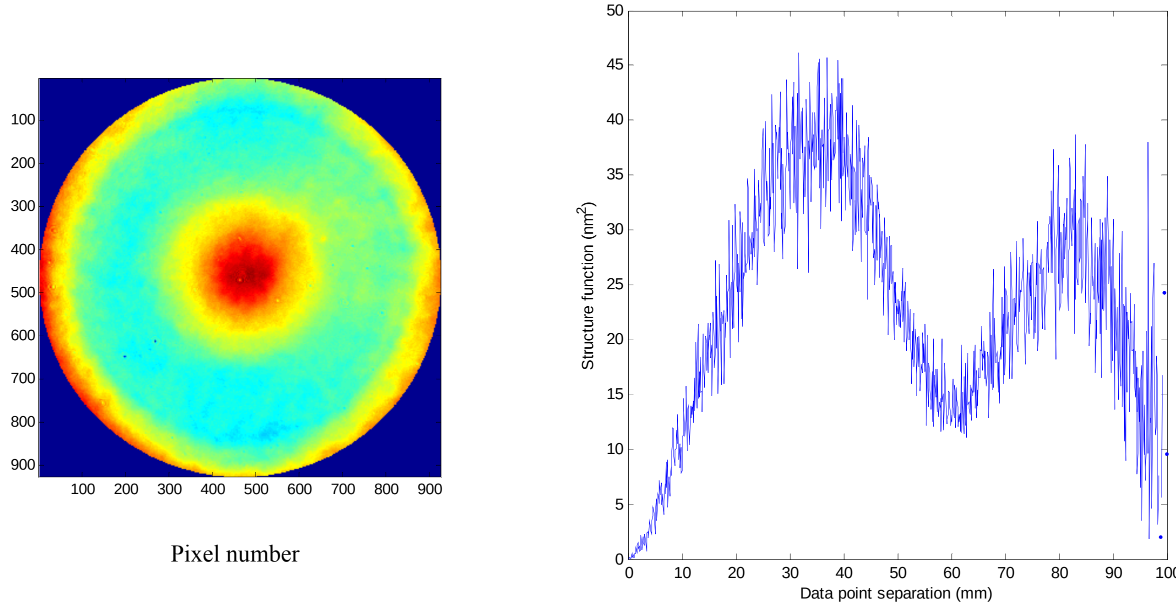

Fig. 4 Contour map (a) of the data used to calculate the structure function (b) using 10^5 random data points. The interferogram was 925 pixels square while the aperture was 100 mm in diameter.

To get a better feel for what the data are telling us and to make the results look more familiar to the surface roughness community, the structure function is plotted in blue on log/log scales in Fig. 5. In addition the rms, or square root of the structure function, is shown in green. Finally we have added the rms slope in red where the slope is calculated by dividing the rms height, or phase error, by the separation, r, of the data points in the aperture.

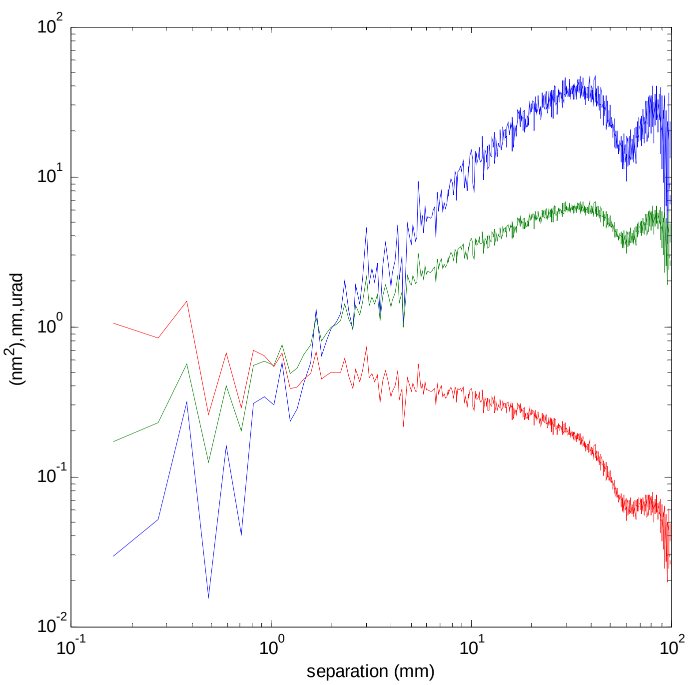

Fig. 5 The structure function of the wavefront in Fig. 4a (blue) plotted on a log/log scale in units of nm^2 along with the rms phase error in nm (green) and the rms slope error in μradians (red)

3.4 Figure versus mid-spatial frequency features

Figure 5 raises a number of interesting issues; for one, what part of the character of the curves is due to traditional figure and what part is mid-spatial frequency roughness. Also, notice that the rms slope increases with decreased data point separation even as the structure function rapidly decreases. The good news about the slope at small spatial separations is that its effect becomes small relative to diffraction effects; that is to say, over a 1 mm area, for example, and a focal length of 100 mm the area on the surface represents an f/100 cone of light with an associated roughly 100 µm spot size that, of course, will have little intensity.

On the figure versus mid-spatial frequency error issue we can explore this by removing low order Zernike terms from the data set and see the effect on the shape of the structure function. By taking out the first 15 terms all the Seidel aberrations are removed and those appear to be a large portion of the wavefront error, the rms error dropping from about 7 nm to 2nm. Fig. 6 shows the wavefront after subtracting the first 15 Zernike terms along with linear and log/log plots of the structure function, rms and rms slope.

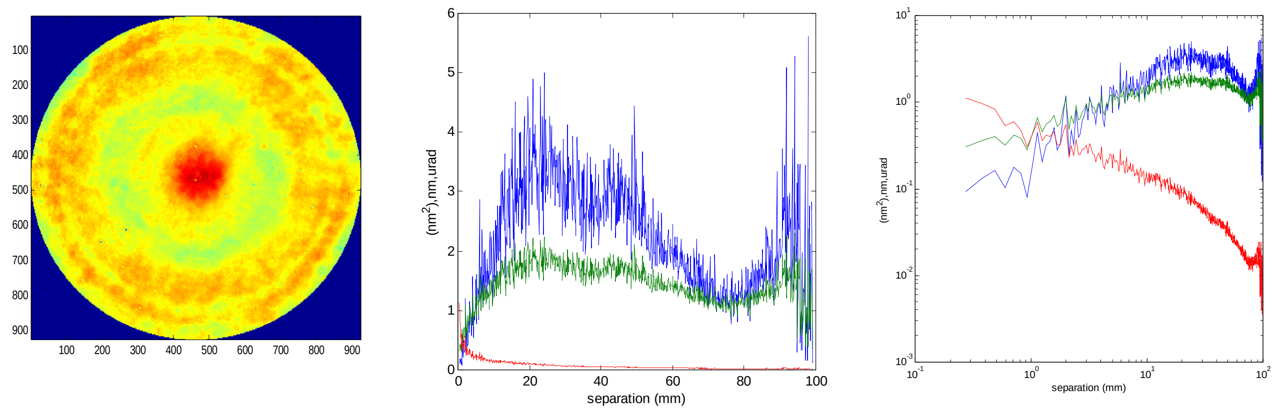

Fig. 6 Residual error after the first 15 Zernike terms were removed from the wavefront in Fig. 4 (left), structure function, rms and slope with linear scales (center) and the same plotted with log/log scales.

The hump in the structure function at 25 mm separation is largely due to the remaining central high in the wavefront dropping to the low zone at a radius of about 20 mm and then the very edge that is again low. The low at 80 mm separation is due to the nearly constant height zone with a radius of perhaps 35 to 47 mm. At this point it could be argued whether or not all the figure error had been removed but that is more of a fabrication than testing issue.

To gain further insight into the break between figure and mid-spatial frequency roughness the first 28 Zernike terms are removed to obtain the results in Fig. 7.

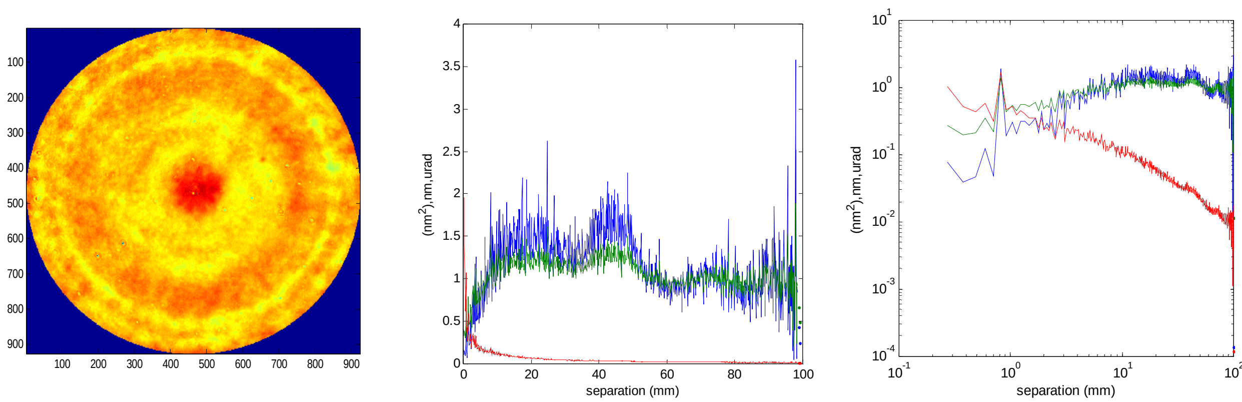

Fig. 7 Residual error after the first 28 Zernike terms were removed from the wavefront in Fig. 4 (left), structure function, rms and slope with linear scales (center) and the same plotted with log/log scales.

Removing the additional 13 terms improved the errors with a separation of about 25 mm and reduced the structure function by about a factor of two. The wavefront is now dominated by an overall choppiness and some high spatial frequency zonal error. We took out the next 17 terms for a total of 45 Zernike terms, but there was little noticeable difference in the wavefront or the structure function.

To a certain degree the division between figure and mid-spatial frequency roughness is more a question of how the optician will attempt to make the surface smoother; this will be influenced by the equipment available and the size of the workpiece. If this were a large piece, the local bump in the middle would still be considered a figure error and gone after by local figuring. On the other hand, if this were a small piece, a full size lap would be used but the hardness of the pitch or texture of the pad changed.

4. STRUCTURE FUNCTION AS A SPECIFICATION

4.1 Criteria for a specification in terms of a structure function

What does all this mean in terms of writing a specification to preclude having images such as that in Fig. 2? It seems that there are three criteria for the specification: what does the figure have to be in the classical sense, where is the division between figure and mid-spatial frequency roughness in terms of aperture size and methods of polishing, and what is a realistic high spatial frequency specification?

As an example, assume we want a 100 mm diameter mirror that will have diffraction limited performance in the visible, that is, have a reflected wavefront that is about $\lambda/14 \approx .07$ waves rms, or about 35 nm rms; this is the first requirement. The second is that anything smaller in scale than 25 mm will be considered mid-spatial frequency roughness rather than figure because the polishing methods do not lend themselves to correct errors with a spatial scale of less than 25mm. Obviously this criteria depends on the particular situation but we have to choose some realistic value. Finally, a finish of 1 nm rms is typical of a good optical polish. Thus a specification plotted in the same manner as the structure function plots in Figs. 5, 6 and 7 would look like that in Fig. 8.

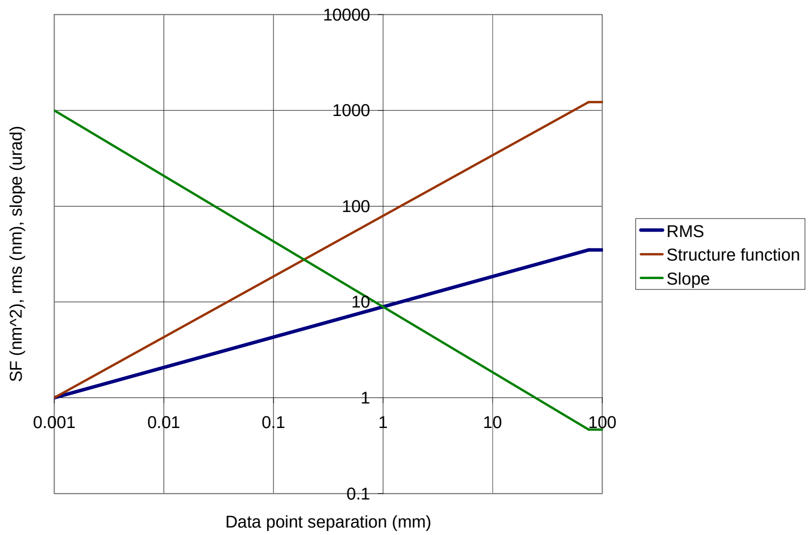

Fig. 8 Structure function, rms and rms slope based on a figure specification of 35 nm rms for separation distances of 25 mm or greater and a finish specification of 1 nm rms

The plot in Fig. 8 shows a greater spatial scale than the other Figures because we wanted to show the finish spec of 1 nm rms at the 1 micron spatial scale that is typical of the minimum spatial scale of surface finish measurements. One thing to notice is that this relatively good finish spec implies a local slope error of 1 mr on the 1 μm scale, ordinarily considered huge but effectively swamped by diffraction. Also note a slope of about .35 μradians over the full aperture, small enough that the image size is also dominated by diffraction. The slight hook (barely visible in the log/log plot) in all three curves is at the 25 mm spatial point separation at which point the specification is constant.

The power law equations of the varying parts of the three curves are structure function = 79.5*r^.633 nm^2, rms = 8.9*r^.317 nm and rms slope = 8.9*r^(.317-1) = 8.9*r^-.683 µrad. Notice that to determine the rms slope the rms is divided by r so the exponent of the rms slope is just that of the rms less 1. This obviously puts constraints on physically realizable specifications regardless of whether a structure function is used or not.[9]

4.2 Application of the structure function specification

To better compare this proposed specification with the wavefront used as an example in Fig. 4a, the plot in Fig. 8 has been spatially scaled to match that in Fig. 5, the structure function, rms and slope of the wavefront of the optic. This is shown in Fig. 9, where the suggested specification is shown along with the data from the part. As in Fig. 4a, only tilt and focus have been removed from the nominally spherical wavefront.

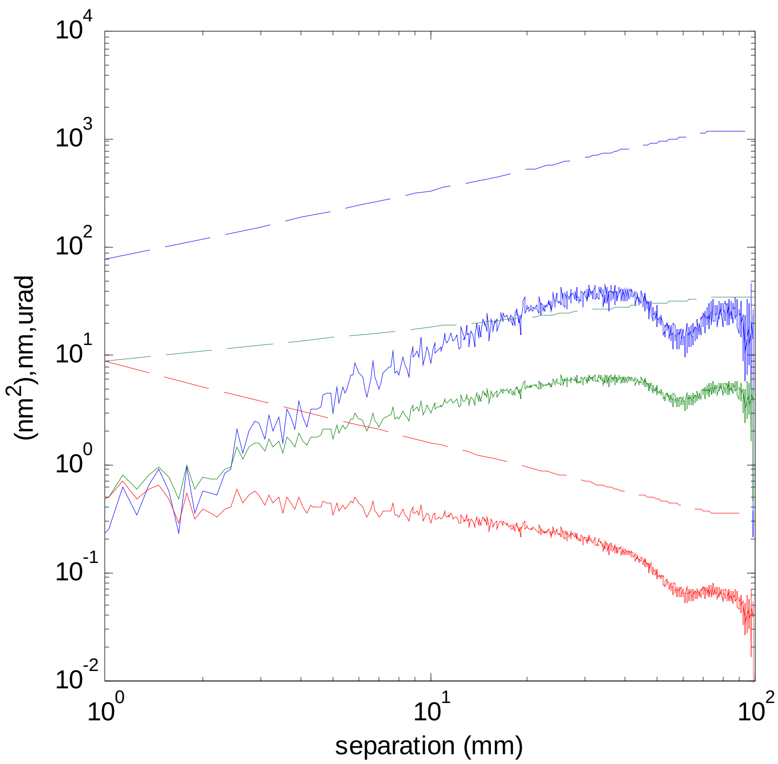

Fig. 9 The structure function, rms and rms slope of the wavefront in Fig. 4a along with the specifications for all three same outlined above.

The part is clearly better than the specification but not by a large margin. We knew from the beginning that the part had an rms figure error of <7 nm rms and since its rms and structure function drop farther below the specification for smaller data point separations the finish is probably less than 1 nm rms on the µm spatial scale. While finish tends to obey a power law we are probably correct in this surmise but until the finish is actually measured we cannot be certain.[10]

5. CONCLUSIONS

We have given several examples of optics that met their explicit specifications yet could be expected to perform poorly because the explicit specification carried an implicit specification in the mind of the customer that was unwarranted. The customer should have given a specification that covered mid-spatial frequency errors as well as figure and finish.

As optics are polished by more and more deterministic methods using small tools or wear functions relative to the size of the optical surface, the likelihood of seeing mid-spatial frequency errors will become greater, thus it becomes more important than ever to specify what is an acceptable phase error in the mid-spatial frequency region. We show it is possible to write a mid-spatial frequency specification relatively simply and show how to measure this error interferometrically by analyzing the resulting phase data using the structure function. We have given an example of a realistic method of developing and applying such a structure function specification.

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