applied optics

# Measuring the four paraxial lens parameters using an autostigmatic microscope

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**We describe a method of measuring the four paraxial lens parameters—the two radii, the center thickness, and the index—of a realistic-size positive lens using an autostigmatic microscope (ASM). The method is similar to measuring the radius of curvature of a concave mirror with an ASM but slightly more complex in that four characteristic distances must be measured to solve for the four unknown parameters. Once the four distances are measured, it is shown how to use an Excel spreadsheet and the add-in iterative "Solver" to find the four unknown parameters. Finding the paraxial lens parameters is useful for troubleshooting a lens assembly that does not perform as expected due to mislabeling, the incorrect glass type used, insertion into the assembly backward, or for finding a replacement glass type.** 

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## 1. INTRODUCTION

Occasionally, it becomes necessary to determine the paraxial lens parameters [1] of a singlet lens because the lens was mislabeled, perhaps inserted in an assembly backward because the two radii were close, the wrong glass was mistakenly used in manufacture, or the lens must be reengineered because the glass type is no longer available. For whatever reason, these four paraxial parameters found in the lens maker's equation must be determined, that is, the two radii, the center thickness, and the index.

This is quite easy to do using an autostigmatic microscope (ASM) [2], which is an optical instrument similar to an autocollimator but with a finite focus. The four paraxial parameters are found in much the same way as when an ASM is used to measure the radius of curvature of a concave mirror [3]. Because most lens elements are positive, we will use this case as an example but also indicate that much the same method works equally well for negative elements with the addition of a concave mirror.

First, we describe an example positive lens and explain why it is a typical example. Then, we indicate the distances, or thicknesses, which have to be measured and give paraxial equations for these derived from the lens maker's equation. We discuss which of the possible measurements are most sensitive and select those as the measurements to perform. Finally, we show how to solve for the unknown parameters. There may be a closed-form solution, but it is straightforward to use a spreadsheet program along with a built-in iterative equation solver to find the desired results.

## 2. EXAMPLE LENS

The positive singlet used to illustrate the procedure has a back focal length (BFL) of 100 mm and a clear aperture of 24 mm and is designed for use at infinite conjugates (see Fig. 1). As such, it is typical of many lenses that might require paraxial parameter determination in that the radii are too long to be within the typical working distance (10–20 mm) of a microscope objective that might be used on an ASM. This means that neither of the lens radii can be measured directly at their centers of curvature as described in [3].

On the other hand, when looking into the lens from either side, the far radius of curvature will appear concave and will be easily accessible by the ASM. Also the BFL is easily measured against a plane mirror because this is a positive lens. Using a Cat's eye reflection off either of the rear surfaces looking in from either side, the apparent center thickness also can be measured, but it is the least-sensitive thickness to use in solving for the required parameters because it is small compared with the other distances.

## 3. PARAXIAL EQUATIONS

The lens, as it would normally be used to focus light from infinity, is shown in Fig. 1 with $R_1$ as the first surface. In making the distance measurements, the ASM is looking at the lens from the right, and, if we were measuring the BFL, the ASM focus would be at the place where the rays focus in Fig. 1. For this discussion, we note that $R_1$ stays with the lens when it is reversed and will always be 65.730 mm, but the sign will change. Also, the

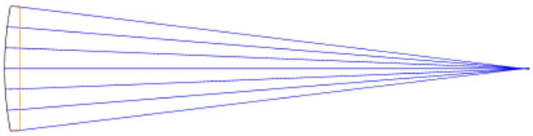

**Fig. 1.** Example lens with a back focal length (BFL) of 100 mm designed for use at infinite conjugates. $R_1 = 65.73$, $R_2 = -841.804$, $t = 3.0$, $n = 1.60$.

zero distance for all measurements will be the surface facing the ASM, $R_2$ in the case of Fig. 1. Distances to the right of $R_2$ are positive.

By paraxial ray tracing, the optical center thickness of the lens is found to be

$$t_o = \frac{-R * t}{[t * (n - 1 + n * R)]}, \tag{1}$$

where $t$ is the physical thickness, $n$ the index, and $R$ the radius of the surface facing the ASM, $R_2$ in Fig. 1 with the sign changed. This gives

$$t_0 = \frac{841.804 * 3}{3 * (1.6 - 1) + 1.6 * -841.804} = -1.878, \tag{2}$$

which is a small number relative to the radii we are trying to determine. This is not much help in solving for the radii but useful as a quick check of center thickness without having to physically touch the lens surface. If the lens were reversed, we would find $t_0 = -1.843$ mm.

Again by paraxial ray tracing the center of curvature of $R_1$ looking into the lens from the right is given as

$$R_{o1} = \frac{-R_2(R_1 - t)}{(R_1 - t)(n - 1) - nR_2} = 38.140 \text{ mm}. \tag{3}$$

If the lens is reversed, we simply exchange $R_1$ and $R_2$ in the formula and change the signs of each as well to find $R_{o2} = 90.6148$ mm. In both cases, these are measured with respect to the surface closest to the ASM.

In a similar manner, it can be shown that the BFL of the lens is given by

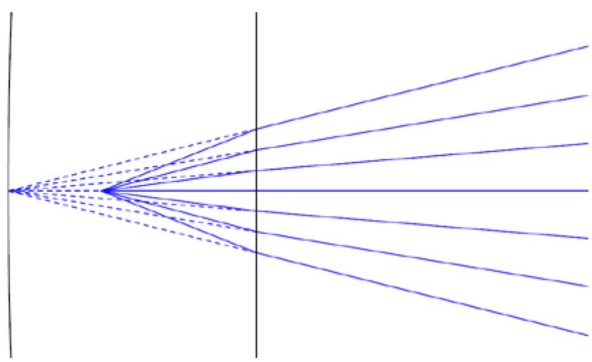

**Fig. 2.** Central part of the lens showing rays coming from the right from the ASM objective focused 1.878 mm into the lens but appearing to come (dashed lines) from the far vertex Cat's eye reflection due to refraction at the surface nearest the objective.

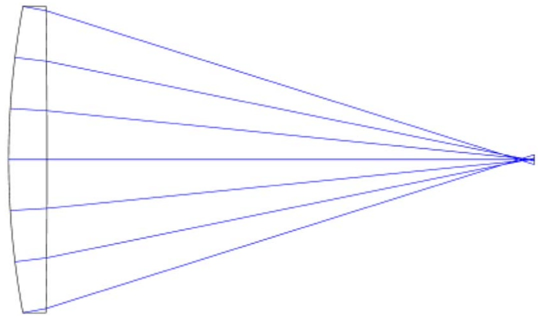

**Fig. 3.** Rays coming from the center of curvature of $R_1$ after refraction in $R_2$ and focusing at $R_{o1} = 38.140$ mm from $R_2$.

$$bfl_{o1} = \frac{R_1[t - n(R_2 + t)]}{(n - 1)[t + nR_1 - n(R_2 + t)]} = 101.606 \text{ mm}. \tag{4}$$

To find $\mathrm{bfl}_{o2}$, we do the same thing: substitute $R_1$ for $R_2$ and change signs of both to get $\mathrm{bfl}_{o2} = 100.000$ mm.

Figures 2 and 3 show the rays for the center thickness and radius of curvature, while Fig. 1 shows the case for $\mathrm{bfl}_{o2}$. In Fig. 2, the ASM is focused at 1.878 mm into the 3 mm thick lens, but the Cat's eye reflection appears to be coming from the vertex of the far surface.

In Fig. 3, the rays are refracted at the surface closest to the ASM, so they reflect from the far surface at normal incidence.

## 4. CALCULATION OF THE LENS PARAMETERS

We now have a total of six thickness measurements, three from each side of the lens. We put these values, the highlighted numbers in yellow, into the Excel spreadsheet, as shown in Fig. 4.

To use the spreadsheet, estimates for the four paraxial lens parameters are entered in the boxes highlighted in green. Immediately below these boxes are calculations of what the six measured thicknesses should be based on the initial

| Lens parameters/calculated or measured values | Initial estimates of lens parameters | | |
|---|---|---|---|
| Estimated value for index, **n** | 1.600 | | |
| Estimated value for physical thickness, **t** | 3.000 | | |
| Estimated value for first surface radius, **R1** | 841.804 | | |
| Estimated value for second surface radius, **R2** | -65.730 | | |
| | Calculated thickness based on lens parameters above | Measured thickness from surface nearest ASM | Squared difference between measured and calculated |
| Thickness from R2 surface to focus, **d1** | 90.615 | 90.615 | 6.65345E-08 |
| Thickness to R1 vertex from R2 surface, **t1** | -1.908 | -1.908 | 1.22885E-12 |
| Reverse, thickness from R1 surface to focus, **d2** | 38.140 | 38.140 | 5.18067E-09 |
| Reverse, thickness to R2 vertex from R1, **t2** | -1.878 | -1.878 | 1.15354E-12 |
| Distance from R2 surface to focus, **bfl1** | 100.000 | 100.000 | 1.32115E-08 |
| Reverse, distance from R1 surface to focus, **bfl2** | 101.606 | 101.606 | 2.49799E-08 |
| Sum of squared differences | | | 1.09909E-07 |

**Fig. 4.** Spreadsheet used to calculate lens parameters from six thickness measurements. Values highlighted in yellow are those obtained by measurement while those to the left are based on first-order calculations. This example shows the results (in green) based on "perfect" measurements.

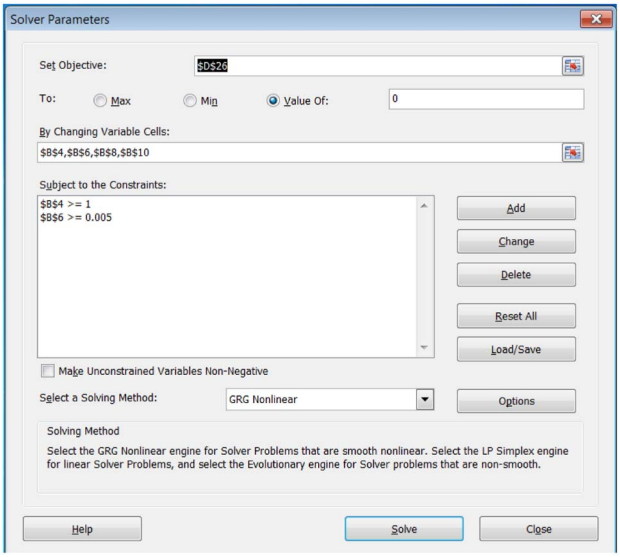


**Fig. 5.** "Solver Parameters" window filled in for the example data shown in Fig. 4.

estimates. The yellow highlighted numbers are filled in based on the measured thicknesses, and there will always be small differences between the calculated and measured values. The square of these differences, to keep the differences positive, are shown in the last column and the sum of the squared differences in the last box (highlighted in orange). Note that the third, fourth, and sixth distances all assume the lens is reversed and that the $R_1$ side is now toward the ASM, which is pointed toward the lens from the right.

To find the four unknown lens parameters (highlighted in green), it is necessary to make the square of the sum of the differences (in orange) as small as possible. To do this, the "Solver" add-on in Excel is used [4]. First, the Solver must be installed in Excel and then the "Solver Parameters" menu set up. To get here, once the Solver is installed, click on the Excel "Data" tab and then "Solver" at the far right.

In the Solver parameter box (see Fig. 5), put the value that is the sum of the squared differences in the "Set Objective" box because this is what we want "To" have a "Value of" 0. We get this "By Changing Variable Cells" with the estimates of the four lens parameters. The "Constraints" are added using the "Add" tab and keep the index $>1$ and the thickness $>$ than some small number. Be sure the "Make Unconstrained Variables Non-Negative" is unchecked because at least one radius will be negative.

In general, a solution will be found using the setup as shown, but a better solution, one with higher precision, can be found by clicking the "Options" box and adding some zeros to the "Constraint Precision" box and under "GRG Nonlinear" adding some zeros to the "Convergence" and checking the "Central" derivatives box. Then hit the "Solve" button, and an answer should appear for the four lens parameters. If the results of the measurements were not too precise, the Solver may come back and say "No Solution Found" because it cannot make the sum of the squares as small as the number of zeros in the "Convergence" box. You can either accept the solution as good enough as it is or loosen the convergence until the Solver finds a solution.

## 5. DISCUSSION

Although the example is for a bi-convex singlet lens, it should be clear that the same basic method applies for any form of positive lens. If the lens is plano–convex, simply use a large number like 1e10 for the plano side in the spreadsheet [5]. It is easy to make an error in sign or get the surfaces reversed in the calculation; thus, common sense must be used if the answers do not seem to match the experimental situation.

For negative lenses, the same approach can be used, but a concave mirror must be used to create a real focus that the ASM can access. The concave sphere must have enough power so the combination of sphere and lens form a positive optical pair. Obviously, the spreadsheet and formulas must be adjusted to take the sphere into account, but the methodology of the process is exactly the same.

## 6. CONCLUSION

It has been shown how to solve for the four paraxial lens parameters of any positive lens by measuring a set of at least four distances so there is sufficient data to solve the set of equations. The distance measurements are similar to those made when measuring the radius of curvature of a concave sphere with an autostigmatic microscope. There does not appear to be a closed-form solution to finding the lens parameters, so a spreadsheet is used along with an iterative equation solver to find the four lens parameters simultaneously.

For those more versed in lens design, the paraxial parameters also can be found using a four (or more to match the situation) configuration design for the measured distances. Here, the lens design optimizer is generally constrained enough that estimates for radii can be plano surfaces and the index and thickness almost any positive values. Again, it is possible to find the paraxial parameters for a negative lens by adding a concave sphere to the test setup to force the pair of optics to produce a real image between the lens and the ASM.

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